JEE Advanced Paper – 2

MATHEMATICS

Sol:41. $\sin x + 2\sin 2x - \sin 3x = 3$

$\Rightarrow \sin_2 x\left(1+2\ \cos x - 3 + 4\sin^2\right) = 3$

$\Rightarrow 4\sin^2 x + 2\ \cos x - 2 = 3/\sin x$

$\Rightarrow 2 - 4\cos^2 x + 2\cos x - 2 = 3/\sin x$

$\Rightarrow 9/4 - \left(2\cos\ x - 1/2\right)^2 = \ 3/\sin x$

$L.H.S \leq 9/4\, R.H.S \geq 3$

(D) No solution

Sol:42. $\int_{45^\circ}^{90^\circ} \left(2\cos ec\, x\right)^{17} dx$

$Put\ 2\ \cos ec\ x = e^4 + e^{-4}$

$2\cos ec \times \omega tx\, dx = \left(\mathrm{e}^4 + \mathrm{e}^{-4}\right) dx$ ....(1)

Wehaur $\cot^2 1 = \cos ec^2 \times x - 1$

$4\cot^2 x = 4\cos ec^2 x - 4$

$\Rightarrow \left(2\cot x\right)^2 = \left(e^4 + e^{-4}\right)^2 - 4$

$\Rightarrow \left(2\cot x\right)^2 = \left(e^4 + e^{-4}\right)^2$

$\Rightarrow 2\cot x = e^4 - e^{-4}$

From eqn (1), $2\cos ec\, x \cot x\, dx = 2\cot x\, dx$

$Dx = 2du/e^4 + e^{-4}$ (2)

$$\therefore \int_{40^\circ}^{90^\circ} (2\cos c\,x)^{17}\,dx = \left(e^4 + e^{-4}\right)^{17} 2dx \big/ \left(e^4 + e^{-4}\right)$$

$$2\left(e^4 + e^{-4}\right)^{16} dx$$

$$E^4 + e^{-4} = 2\cos e\,x$$

$$E^4 - e^{-4} = 2\cot x$$

$$2e^4 = 2\cos ec\,x + 2\cot x$$

$$e^4 \cos ec\,x + \cot x$$

$$\text{at } x = 45^\circ \quad 4 = \text{In}\left(j2+1\right)$$
$$\text{at } x = 90^\circ \quad 4 = \text{In}\left(0+1\right) = 0$$

Sol:43. $p(x) = ax^2 + bx + c$ Since roots are purely imaginary

$$b = 0\left(asx = \text{-}b + \sqrt{b^2 - 4ac/2a}\right)$$
$$p(x) = ax^2 + c$$

And $-4ac < 0$ (roots are imaginary $d < 0$)

$$4ac > 0$$
$$Ac > 0$$
$$P\left(p(x)\right) = 2\left[2x^2 + tc\right]^2 + c$$
$$= a\left[a^2x^4 + c^2 + 2acx^2\right] + c$$
$$= a^3x^4 + 2a^2cx^2 + \left(ac^2 + c\right)$$
$$D = 4a^4c^2 - 4a^3\left(ac^2 + c\right)$$
$$= -4a^2\left(ac\right) < 0$$
$$X^2 = -2a^2C + \sqrt{-4a^2ac}/3\left(ac > 0\right)$$

So all roots are imaginary

Sol:44. $dx/dx + x4/\sqrt{}\,x^2 - 1 = x^4 + 2x/\sqrt{}\,1 - x^2 \quad (co) = 0$

$$\sqrt{}\,x/\sqrt{}\,x^2\,1dx$$
$$I.F.E = \sqrt{}\,x^2 - 1$$

So solution

$$4.\sqrt{}\,x^2 - 1\left|x^4 + 2x/\sqrt{}\,1 - x^2.\sqrt{}\,x^2 - 1dx\right.$$
$$\left|i\left(x^4 + 2x\right)dx + c\right.$$
$$4\sqrt{}\,x^2 - 1 = i\left(x5/5 + x^2\right) + c$$
$$\text{Now } f(0) = 0$$
$$4\sqrt{}\,0^2 - 1 = i(0 + 0) + c$$
$$\Rightarrow [c = 0]$$
$$\therefore 4 = i\left(x5/5 + x^2\right)/\sqrt{}\,x^2 - 1$$
$$\text{So}, I = \int_{53/2}^{j3/2} \frac{\lambda\left(\frac{x^5}{5} + x^2\right)dx}{\lambda\sqrt{1 - x}^2} = \int \underbrace{x^5\,dx/5\sqrt{}\,1 - x^2}_{odd} = \int \underbrace{\frac{x^2dx}{\sqrt{1 - x}^2}}_{Even}$$
$$\Rightarrow I = 2\int_0^{\sqrt{3}/2} \frac{x^2dx}{\sqrt{1 - x^2}}$$
$$\text{Now } x = \sin\,\theta$$
$$I = 2\int_0^{f\,0} \frac{\sin^2\theta}{\cos\theta}\cos d\theta$$
$$= 2\int \sin^2\theta d\theta = \int f\,2\left(1 - \cos\theta/x\right)d\theta$$
$$= \left[\left(\theta - \sin 2\theta/2\right)\right]_0^{60}$$
$$= \left(\pi/3 - \sin 2\pi/3\right) - (0)$$
$$= \pi/3 - 3/4$$

Ans (B)

Sol:45. $\left(1+x^{2}\right)^{4}\left(1+x^{3}\right)^{7}\left(1+x^{4}\right)^{12}$ Coefficient of x^{11} will come from

Coefficient of $\left[\left(x^{2}\right)^{4}\left(x^{3}\right)^{1}+\left(x^{2}\right)^{1}\left(x^{3}\right)^{3}+\left(x^{2}\right)\left(x^{3}\right)^{1}\left(x^{4}\right)^{1}+\left(x^{3}\left(x^{4}\right)^{2}\right]$

$={}^{4}C_{4}{}^{7}C_{1}+{}^{4}C_{1}{}^{7}C_{3}+{}^{4}C_{2}+{}^{7}C_{1}^{12}C_{1}+{}^{7}C_{1}^{12}C_{2}$
$=1\times7+4\times7\times6\times5/3\times2/4\times3/2\times7\times12+7\times12\times11/2$

On solving we get coefficient 1113

Sol:46. $F(x)\int_{0}^{x^{2}} f(\sqrt{t})dt$

Using iebinity formula differing both sides wrt x

$F(x)=2\left(x^{2}\right)/2xf\left(\sqrt{x^{2}}\right)+2(0)/2f(50)$
$f'(x)=f'(x)=2xxf(x)$
$f'(x)=2xf(x)$
$f(x)=2x$
$f(x)$

$f(x)/(x)dx=2x\,dn$

integration both sides

in $f(x)=x^{2}$
$f(x)=e^{x^{2}}$
$f(x)=\int_{0}^{x^{2}} e^{t}at=e^{x^{2}}-1$
$\left[f(2)=e^{4}-1\right]$

Sol:47

$\Rightarrow[1/r=\cot\theta]-(1)$
Area of trapezium $=(rs+pq)/2(AB)$

$=\left(2x^{2}\omega sq\right)\left(4r+\sin\theta\right)$

$=\left(2r^{2}-\sqrt{21}/\sqrt{r^{2}}+1\right)\left(\sqrt{2}r/\sqrt{r^{2}}-+1\right)$

Check for $r=1$

$=3\times5$

$=[15]$

Ans (D)

Sol:48.

cards	Envelops
1	1
2	2
3	3
4	4
5	5
6	6

$\rightarrow$ if '2' goes in '1' then it is derangement of 4 things which can be done in $41\ \left(1/21\text{-}1/31+1/41\right)=9$ ways

$\rightarrow$ if '2' doesn't go in 1 it is derangement of 5 thingswhich can be done in 44 ways

$\rightarrow$ hence total 53 ways

Option (c) is correct

Sol:51. Pl$at^{2}, 2at$

$Q\left(aq^{2},2aq\right)$

$P\left(a,2a\right)\quad K\left(2a,0\right)$

$Q\left(a,2a\right)\quad R\left(ar^{2},2ar\right)$

Slope of $pk=$ *slope of* QR

$0-2a/2a-a=2ar+2a/ar^2-a$

$-2a/a=2a(r+1)/a(r_2-1)$

$R^2-1=-r-\lambda$

$r^2+r=0$

$r=0,\text{-}1 but\ 2c\neq 0$

$\therefore r=-1$

Now $t=1 so\ r=\text{-}1/t\ is\ correct$

Sol:52. eqn of normal: $y=\text{-}5x+2as+as^3$

Eqn of tangent: $x=ty-at^2$

$Y=-5(ty-at^2)+2as+as^3$

$Y=-sty+ast^2+2as+as^3$

Now $st=1\Rightarrow y=-y+a++2as+as^3$

$\Rightarrow 2y=a|t+1/t+1/t^3$

$\Rightarrow y=a(1+t^2)^2/2+3$

So, (B) is correct

Sol:53. $\lim\limits_{h\to 0}\int_h^{rh} t^{-a}(1-t)^{a-1}\,at$

$Tt\,h\to 1$

$h\to 0$

$g(1/2)\int_0^1 t^{-1/2}(1-t)^{-1/2}\,att$

$\int_0^1 1/\sqrt{t}\ \sqrt{}1-t\,dt$

$T=\sin^2 xt\to 0\sin x\to 0$

$dt=2\sin x\cos x\,ds\,t\to 0\,x\to\pi/2$

$$\int_0^{\pi/2} \frac{2\sin x\cos x dx}{\sin x\cos x}$$

$$=2$$

$$\int_0^{\pi/2} dx = 2x\,\pi/2 = \pi$$

Sol:54. $g(a) = \lim_{h\to 0^+} \int_0^{1-h} t^{-a}(1-t)^{a-1}\,dt$

$$2g(a)/2a = 2/2a(1-h)t^{-(1-h)}(1-t)^{(1-h)-1}$$

$$+2(0)/2a\,t^{-0}(1-t)^{0-1}$$

$g^{(1/2)} = 0$ using Leibnitz therein

Sol:55.

box1		box2	box3
$(I)\,odd$	$<$	$even$	$-even$
		odd	$-odd$
$(II)\,even$	$<$	$even$	$-odd$
		odd	$-even$

$$2/3\left[2/5.3/7+3/5.4/7\right]=36/105$$

$$1/3\left[2/5.4/7+3/5.3/7\right]=17/105$$

$$Total = 53/105$$

Sol:56. $2x_2 = x_1 + x_3$

Possible	x_2	x_1	x_3
	1	1	1
	2	1	3
	2	2	2
	2	3	1
	3	1	5
	3	2	4
	3	3	3
	4	1	7
	4	2	6
	4	3	5
	5	3	7

$(Ax1) = 1/3\, p(x_2) = 1/5\, p(x_3) = 1/7$ and there are II cases

$\Rightarrow 11 \times [1/3 \times 1/5 \times 1/7] = 11/105$

Sol:57. $\sum_{k=1}^{9} as(25\pi/10) = \cos(2\pi/10) + \cos(9\pi/10) + \cos(6\pi/10) + \cos(8\pi/10) + \omega^\circ(10\pi/10)$

$+\cos(12\pi/10) + \cos(14\pi/10) + \cos(16\pi/10) + \cos(18\pi/10)$

Now $= \cos(18\pi/10)\cos(2\pi - 18\pi/10) = \cos(2\pi/10)$

Similarity cost $\cos(16\pi/10) = \cos(4\pi/10)$

$\cos(11\pi/10) = \cos(6\pi/10)$

$\cos(12\pi/10) = \cos(8\pi/10)$

$\therefore \sum_{k=1}^{9} \cos(2k\pi/10) = 2[\cos(4\pi/10) + \cos(6\pi/10) + \cos(8\pi/10)] + \cos\pi$

$= 2\underbrace{[2\sin\pi\sin(6\pi/10)]}_{=0} + \underbrace{2\sin\pi\sin(2\pi/10)}_{=0} + -1$

$=-1$

$\therefore -\sum_{k=1}^{9} \cos(2k\pi/10) = 2$

$\therefore s \to 4$

$(Q)\, z_1 z = z_k$

$e^{i(2k\pi/10)} e^{i(2k\pi/10)} = e^{i(2k\pi/10)}$

$1+n = k$ for $k - 1, 2, \dots 9$
$N = 0, 1, \dots 8$

$\therefore (Q)$ is false

This is (C)

Sol:58. $f_2 = x_2 \to$ continues and one – one

$\therefore s \to 4$
$f_3 \to$

f_3 differentiable but not one –one

Sol:59. $(p)\, y = 4x^3 + 3x$ where $\cos\theta = x$

$Dy/dx = 12x^2 - 3$
$d^2/dx^2 + xdy/dx = (x^2 - 1)24x = x(12x^2 - 3)$
$= 36x^3 - 27x = 9(4x^3 - 3x) = 9y$
Hence, $1/y\{(x^2-1)d^2y/dx^2 + x\,dy/dx\} = 9$

(R) Equation of normal $6x/h - 3y/1 = 3$ (Equation of normal is
$a^3x/x - b^2y/y_1 = a^2 - b^2$

Slope $= 6/3h = 1$ cos it is perpendicular to $x + y = 1$

$\Rightarrow R = 2$

Sol:60. $Q =$

$$f(x) = \sin(x^2) + \cos(x^2)$$
$$x \in [-\sqrt{3}, \sqrt{13}]$$
$$x^2 \in [0,13]$$
$$\text{let } x^2 = t$$
$$\Rightarrow t \in [0,13]$$

$$F(x) \sin t + \cos t$$
$$f(x) = 52 \sin(\pi/4 + t)$$

it is max when

$$\pi/4 + t = \pi/2\pi$$
$$\sin(\pi/4 + t) = \sin(\pi/2)$$
$$\pi/4 + t = n\pi + (-1) + (-1)^n \pi/2$$
$$\pi/4 + t = n\pi + (-1)^n \pi/2$$

for $n = 1$

$$\pi/4 + t = \pi - \pi/2$$
$$t = \pi/4 \text{ also } \pi - \pi/4$$
$$\text{i.e. } t = 3\pi/4 \text{ will satisfy}$$

for $n = 2$

$$\pi/4 + t = 2\pi + \pi/2$$
$$t = 2\pi + \pi/4 + 9\pi/4$$

for $n = 3$

$$\pi/4 + t = 3\pi - \pi/2 = 3\pi - \pi/2 - \pi/4$$

12π-

Also $t = 2\pi + (\pi - \pi/4) = 11\pi/4$ will satisfy

So 4 solution in the interval $[0,13]$