

JEE Advanced Paper – 2

MATHEMATICS

Sol:41. $\sin x + 2 \sin 2x - \sin 3x = 3$

$$\Rightarrow \sin_2 x (1 + 2 \cos x - 3 + 4 \sin^2 x) = 3$$

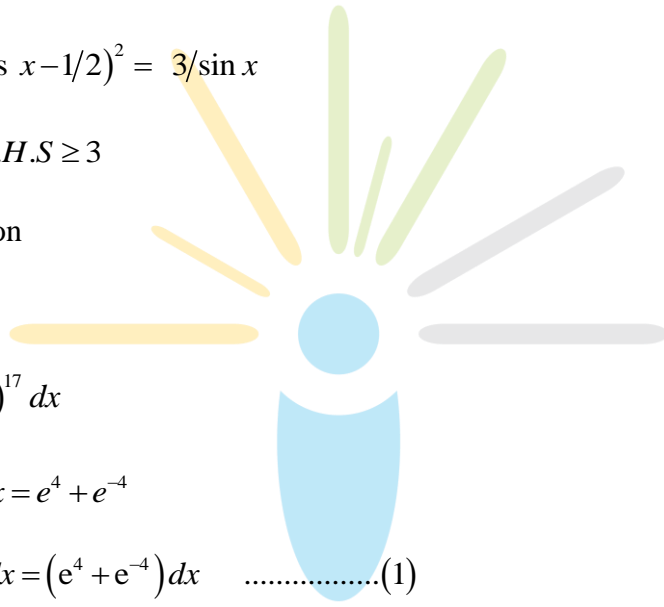
$$\Rightarrow 4 \sin^2 x + 2 \cos x - 2 = 3 / \sin x$$

$$\Rightarrow 2 - 4 \cos^2 x + 2 \cos x - 2 = 3 / \sin x$$

$$\Rightarrow 9/4 - (2 \cos x - 1/2)^2 = 3 / \sin x$$

$$L.H.S \leq 9/4 \quad R.H.S \geq 3$$

(D) No solution



Sol:42. $\int_{45^\circ}^{90^\circ} (2 \operatorname{cosec} x)^{17} dx$

Put $2 \operatorname{cosec} x = e^4 + e^{-4}$

$$2 \operatorname{cosec} x \cot x dx = (e^4 + e^{-4}) dx \dots\dots\dots(1)$$

We have $\cot^2 x = \operatorname{cosec}^2 x - 1$

$$4 \cot^2 x = 4 \operatorname{cosec}^2 x - 4$$

$$\Rightarrow (2 \cot x)^2 = (e^4 + e^{-4})^2 - 4$$

$$\Rightarrow (2 \cot x)^2 = (e^4 + e^{-4})^2$$

$$\Rightarrow 2 \cot x = e^4 - e^{-4}$$

From eqn (1), $2 \operatorname{cosec} x \cot x dx = 2 \cot x dx$

$$Dx = 2du / e^4 + e^{-4} \quad (2)$$

$$\therefore \int_{40^\circ}^{90^\circ} (2 \cos c x)^{17} dx = (e^4 + e^{-4})^{17} 2dx / (e^4 + e^{-4})$$

$$2(e^4 + e^{-4})^{16} dx$$

$$E^4 + e^{-4} = 2 \cos e x$$

$$E^4 - e^{-4} = 2 \cot x$$

$$2e^4 = 2 \cos ec x + 2 \cot x$$

$$e^4 \cos ec x + \cot x$$

$$\text{at } x = 45^\circ \quad 4 = \ln(j2+1)$$

$$\text{at } x = 90^\circ \quad 4 = \ln(0+1) = 0$$

Sol:43. $p(x) = ax^2 + bx + c$ Since roots are purely imaginary

$$b = 0 \left(ax = -b + \sqrt{b^2 - 4ac} / 2a \right)$$

$$p(x) = ax^2 + c$$

And $-4ac < 0$ (roots are imaginary $d < 0$)

$$4ac > 0$$

$$Ac > 0$$

$$P(p(x)) = 2[2x^2 + tc]^2 + c$$

$$= a[a^2x^4 + c^2 + 2acx^2] + c$$

$$= a^3x^4 + 2a^2cx^2 + (ac^2 + c)$$

$$D = 4a^4c^2 - 4a^3(ac^2 + c)$$

$$= -4a^2(ac) < 0$$

$$X^2 = -2a^2C + \sqrt{-4a^2ac/3}(ac > 0)$$

So all roots are imaginary

Sol:44. $\frac{dx}{\sqrt{x^2-1}} + x^4/\sqrt{x^2-1} = x^4 + 2x/\sqrt{1-x^2} \quad (co) = 0$

$$\sqrt{x}/\sqrt{x^2-1} dx$$

$$I.F.E = \sqrt{x^2-1}$$

So solution

$$4.\sqrt{x^2-1} \left[x^4 + 2x/\sqrt{1-x^2} \cdot \sqrt{x^2-1} dx \right]$$

$$\left| i(x^4 + 2x) dx + c \right.$$

$$4\sqrt{x^2-1} = i(x^5/5 + x^2) + c$$

$$\text{Now } f(0) = 0$$

$$4\sqrt{0^2-1} = i(0+0) + c$$

$$\Rightarrow [c = 0]$$

$$\therefore 4 = i(x^5/5 + x^2)/\sqrt{x^2-1}$$

$$\text{So, } I = \int_{53/2}^{j3/2} \frac{\lambda \left(\frac{x^5}{5} + x^2 \right) dx}{\lambda \sqrt{1-x^2}} = \int \underbrace{x^5 dx/5}_{\text{odd}} \sqrt{1-x^2} = \int \frac{x^2 dx}{\underbrace{\sqrt{1-x^2}}_{\text{Even}}}$$

$$\Rightarrow I = 2 \int_0^{\sqrt{3}/2} \frac{x^2 dx}{\sqrt{1-x^2}}$$

$$\text{Now } x = \sin \theta$$

$$I = 2 \int_0^{\pi/3} \frac{\sin^2 \theta}{\cos \theta} \cos \theta d\theta$$

$$= 2 \int \sin^2 \theta d\theta = \int 2(1 - \cos \theta/x) d\theta$$

$$= \left[(\theta - \sin 2\theta/2) \right]_0^{\pi/3}$$

$$= (\pi/3 - \sin 2\pi/3) - (0)$$

$$= \pi/3 - 3/4$$

Ans (B)

Sol:45. $(1+x^2)^4 (1+x^3)^7 (1+x^4)^{12}$ Coefficient of x^{11} will come from

$$\text{Coefficient of } \left[(x^2)^4 (x^3)^1 + (x^2)^1 (x^3)^3 + (x^2)(x^3)^1 (x^4)^1 + (x^3 (x^4)^2 \right]$$

$$= {}^4 C_4 {}^7 C_1 + {}^4 C_1 {}^7 C_3 + {}^4 C_2 + {}^7 C_1 {}^{12} C_1 + {}^7 C_1 {}^{12} C_2$$

$$= 1 \times 7 + 4 \times 7 \times 6 \times 5 / 3 \times 2 / 4 \times 3 / 2 \times 7 \times 12 + 7 \times 12 \times 11 / 2$$

On solving we get coefficient 1113

Sol:46. $F(x) \int_0^{x^2} f(\sqrt{t}) dt$

Using Leibnitz formula differentiating both sides wrt x

$$F(x) = 2(x^2) / 2x f(\sqrt{x^2}) + 2(0) / 2 f(50)$$

$$f'(x) = f'(x) = 2x f(x)$$

$$f'(x) = 2x f(x)$$

$$f(x) = 2x$$

$$f(x)$$

$$f(x) / (x) dx = 2x dx$$

integration both sides

$$\ln f(x) = x^2$$

$$f(x) = e^{x^2}$$

$$f(x) = \int_0^{x^2} e^{at} dt = e^{x^2} - 1$$

$$[f(2) = e^4 - 1]$$

Sol:47

$$\Rightarrow [1/r = \cot \theta] - (1)$$

$$\text{Area of trapezium} = (rs + pq) / 2 (AB)$$

$$= (2x^2 \cos q)(4r + \sin \theta)$$

$$= (2r^2 - \sqrt{21}/\sqrt{r^2 + 1})(\sqrt{2r}/\sqrt{r^2 - 1})$$

Check for $r = 1$

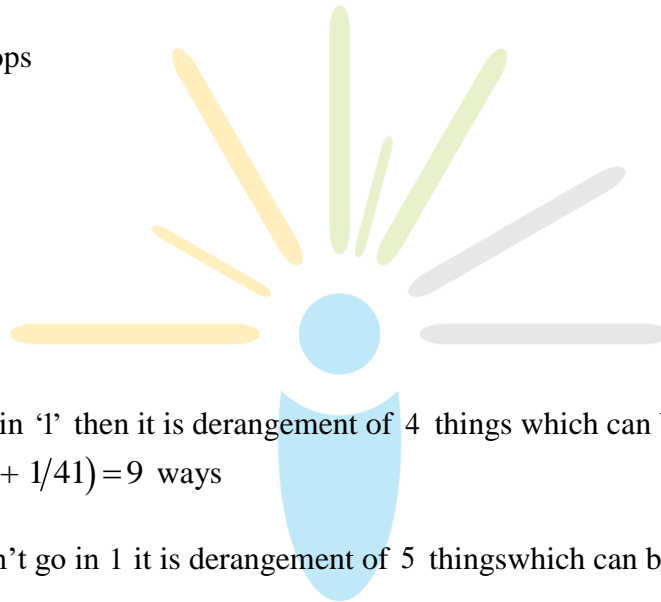
$$= 3 \times 5$$

$$= [15]$$

Ans (D)

Sol:48.

cards	Envelops
1	1
2	2
3	3
4	4
5	5
6	6



→ if '2' goes in '1' then it is derangement of 4 things which can be done in 4! $(1/2! - 1/3! + 1/4!) = 9$ ways

→ if '2' doesn't go in 1 it is derangement of 5 things which can be done in 44 ways

→ hence total 53 ways

Option (c) is correct

Sol:51. $Plat^2, 2at$

$$Q(aq^2, 2aq)$$

$$P(a, 2a) \quad K(2a, 0)$$

$$Q(a, 2a) \quad R(ar^2, 2ar)$$

Slope of $pk = \text{slope of } QR$

$$0 - 2a/2a - a = 2ar + 2a/ar^2 - a$$

$$-2a/a = 2a(r+1)/a(r_2 - 1)$$

$$R^2 - 1 = -r - \lambda$$

$$r^2 + r = 0$$

$$r = 0, -1 \text{ but } 2c \neq 0$$

$$\therefore r = -1$$

Now $t = 1$ so $r = -1/t$ is correct

Sol:52. eqn of normal: $y = -5x + 2as + as^3$

$$\text{Eqn of tangent: } x = ty - at^2$$

$$Y = -5(ty - at^2) + 2as + as^3$$

$$Y = -sty + ast^2 + 2as + as^3$$

$$\text{Now } st = 1 \Rightarrow y = -y + a + 2as + as^3$$

$$\Rightarrow 2y = a \left(t + 1/t + 1/t^3 \right)$$

$$\Rightarrow y = a(1+t^2)^2 / 2 + 3$$

So, (B) is correct

Sol:53. $\lim_{h \rightarrow 0} \int_h^{1+h} t^{-a} (1-t)^{a-1} dt$

$$T \text{ as } h \rightarrow 1$$

$$h \rightarrow 0$$

$$g(1/2) \int_0^1 t^{-1/2} (1-t)^{-1/2} dt$$

$$\int_0^1 1/\sqrt{t} \sqrt{1-t} dt$$

$$T = \sin^2 x \rightarrow 0 \sin x \rightarrow 0$$

$$dt = 2 \sin x \cos x dx \rightarrow 0 \rightarrow \pi/2$$

$$\int_0^{\pi/2} \frac{2 \sin x \cos x dx}{\sin x \cos x}$$

$$= 2$$

$$\int_0^{\pi/2} dx = 2x \pi/2 = \pi$$

Sol:54. $g(a) = \lim_{h \rightarrow 0^+} \int_0^{1-h} t^{-a} (1-t)^{a-1} dt$

$$2g(a)/2a = 2/2a(1-h)t^{-(1-h)}(1-t)^{(1-h)-1}$$

$$+ 2(0)/2at^{-0}(1-t)^{0-1}$$

$g^{(1/2)} = 0$ using Leibnitz therein

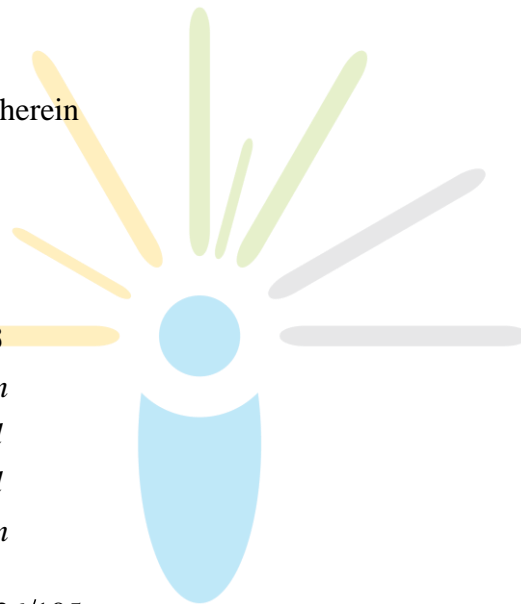
Sol:55.

	box1	box2	box3
(I) odd	<	even	-even
		odd	-odd
(II) even	<	even	-odd
		odd	-even

$$2/3[2/5.3/7 + 3/5.4/7] = 36/105$$

$$1/3[2/5.4/7 + 3/5.3/7] = 17/105$$

$$\text{Total} = 53/105$$



Sol:56. $2x_2 = x_1 + x_3$

Possible	x_2	x_1	x_3
	1	1	1
	2	1	3
	2	2	2
	2	3	1
	3	1	5
	3	2	4
	3	3	3
	4	1	7
	4	2	6
	4	3	5
	5	3	7

$(Ax1) = 1/3 p(x_2) = 1/5 p(x_3) = 1/7$ and there are 11 cases

$$\Rightarrow 11 \times [1/3 \times 1/5 \times 1/7] = 11/105$$

Sol:57. $\sum_{k=1}^9 \cos(2k\pi/10) = \cos(2\pi/10) + \cos(4\pi/10) + \cos(6\pi/10) + \cos(8\pi/10) + \cos(10\pi/10)$

$$+ \cos(12\pi/10) + \cos(14\pi/10) + \cos(16\pi/10) + \cos(18\pi/10)$$

$$\text{Now} = \cos(18\pi/10)\cos(2\pi - 18\pi/10) = \cos(2\pi/10)$$

$$\text{Similarity cost } \cos(16\pi/10) = \cos(4\pi/10)$$

$$\cos(14\pi/10) = \cos(6\pi/10)$$

$$\cos(12\pi/10) = \cos(8\pi/10)$$

$$\therefore \sum_{k=1}^9 \cos(2k\pi/10) = 2[\cos(4\pi/10) + \cos(6\pi/10) + \cos(8\pi/10)] + \cos \pi$$

$$= 2 \left[\underbrace{2 \sin \pi \sin(6\pi/10)}_{=0} \right] + 2 \left[\underbrace{\sin \pi \sin(2\pi/10)}_{=0} \right] + -1$$

$$= -1$$

$$\therefore -\sum_{k=1}^9 \cos(2k\pi/10) = 2$$

$$\therefore s \rightarrow 4$$

$$(Q) z_1 z = z_k$$

$$e^{i(2k\pi/10)} e^{i(2k\pi/10)} = e^{i(2k\pi/10)}$$

$$1+n = k \text{ for } k = 1, 2, \dots, 9$$

$$N = 0, 1, \dots, 8$$

$\therefore (Q)$ is false

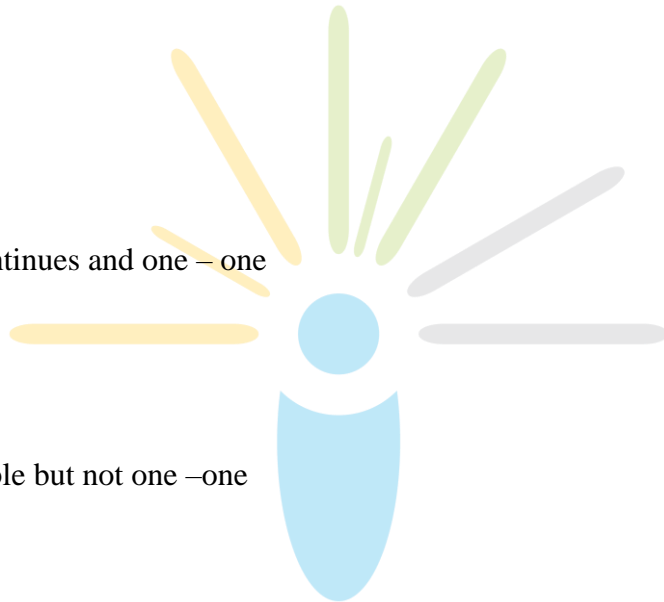
This is (C)

Sol:58. $f_2 = x_2 \rightarrow$ continues and one – one

$$\therefore s \rightarrow 4$$

$$f_3 \rightarrow$$

f_3 differentiable but not one – one



Sol:59. $(p) y = 4x^3 + 3x$ where $\cos \theta = x$

$$Dy/dx = 12x^2 - 3$$

$$d^2/dx^2 + xdy/dx = (x^2 - 1)24x = x(12x^2 - 3)$$

$$= 36x^3 - 27x = 9(4x^3 - 3x) = 9y$$

$$\text{Hence, } 1/y \{ (x^2 - 1)d^2y/dx^2 + xdy/dx \} = 9$$

(R) Equation of normal $6x/h - 3y/1 = 3$ (Equation of normal is

$$a^3x/x - b^2y/y_1 = a^2 - b^2$$

Slope = $6/3h = 1$ cos it is perpendicular to $x + y = 1$

$$\Rightarrow R = 2$$

Sol:60. $Q =$

$$f(x) = \sin(x^2) + \cos(x^2)$$

$$x \in [-\sqrt{3}, \sqrt{13}]$$

$$x^2 \in [0, 13]$$

$$\text{let } x^2 = t$$

$$\Rightarrow t \in [0, 13]$$

$$F(x) \sin t + \cos t$$

$$f(x) = 52 \sin(\pi/4 + t)$$

it is max when

$$\pi/4 + t = \pi/2$$

$$\sin(\pi/4 + t) = \sin(\pi/2)$$

$$\pi/4 + t = n\pi + (-1)^n \pi/2$$

$$\pi/4 + t = n\pi + (-1)^n \pi/2$$

for $n = 1$

$$\pi/4 + t = \pi - \pi/2$$

$$t = \pi/4 \text{ also } \pi - \pi/4$$

i.e. $t = 3\pi/4$ will satisfy

for $n = 2$

$$\pi/4 + t = 2\pi + \pi/2$$

$$t = 2\pi + \pi/4 + 9\pi/4$$

for $n = 3$

$$\pi/4 + t = 3\pi - \pi/2 = 3\pi - \pi/2 - \pi/4$$

$$12\pi -$$

Also $t = 2\pi + (\pi - \pi/4) = 11\pi/4$ will satisfy

So 4 solution in the interval $[0, 13]$

